



Analytical Geometry

Question 1 a)

Lets first find each of the side lengths to determine if the triangle is **obtuse**, **acute** or scalene.

$$\overline{AB} = \sqrt{(-1 - 7)^2 + (5 - 2)^2} = \sqrt{64 + 9} = \sqrt{73}$$

$$\overline{BC} = \sqrt{(7 - (-1))^2 + (2 - (-4))^2} = \sqrt{64 + 36} = \sqrt{100} = 10$$

$$\overline{AC} = \sqrt{(-1 - (-1))^2 + (5 - (-4))^2} = \sqrt{0^2 + 9^2} = \sqrt{81} = 9$$

$$\therefore \overline{AB} \neq \overline{BC} \neq \overline{AC}$$

$\therefore \triangle ABC$ is a scalene triangle.

Question 1 b)

The **orthocenter** is the POI of the heights of a triangle.

$$m_{AB} = \frac{2 - 5}{7 - (-1)} = \frac{-3}{8}$$

$$m_{\perp AB} = \frac{8}{3}$$

$$y_{\perp AB} - (-4) = \frac{8}{3}(x - (-1)) \implies y_{\perp AB} + 4 = \frac{8}{3}(x + 1)$$

$$y_{\perp AB} = \frac{8}{3}x + \frac{8}{3} - 4$$

$$y_{\perp AB} = \frac{8}{3}x - \frac{4}{3} \quad (1)$$

$$m_{BC} = \frac{2 - (-4)}{7 - (-1)} = \frac{6}{8} = \frac{3}{4}$$

$$m_{\perp BC} = \frac{-4}{3}$$

$$y_{\perp BC} - 5 = \frac{-4}{3}(x - (-1)) \implies y_{\perp BC} - 5 = \frac{-4}{3}(x + 1)$$

$$y_{\perp BC} = \frac{-4}{3}x - \frac{4}{3} + 5$$

$$y_{\perp BC} = \frac{-4}{3}x + \frac{11}{3}$$

$$\begin{cases} y_{\perp AB} = \frac{8}{3}x - \frac{4}{3} & (1) \\ y_{\perp BC} = \frac{-4}{3}x + \frac{11}{3} & (2) \end{cases}$$

Sub (1) into (2):

$$\frac{8}{3}x - \frac{4}{3} = \frac{-4}{3} + \frac{11}{3}$$

$$8x - 4 = -4x + 11$$

$$12x = 15$$

$$x = \frac{5}{4} \quad (3)$$

Sub (3) into (2)

$$y = \frac{-20}{12} + \frac{11}{3}$$

$$y = \frac{-5}{3} + \frac{11}{3}$$

$$y = \frac{6}{3} = 2$$

$$y = 2$$

∴ The **orthocenter** is at $(\frac{5}{4}, 2)$

Question 2 a)

$$\text{midpoint} = \left(\frac{\sqrt{72} + \sqrt{32}}{2}, \frac{-\sqrt{12} - \sqrt{48}}{2} \right)$$

$$= \left(\frac{6\sqrt{2} + 4\sqrt{2}}{2}, \frac{-2\sqrt{3} - 4\sqrt{3}}{2} \right)$$

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$$= (5\sqrt{2}, -3\sqrt{3})$$

∴ The midpoint is at $(5\sqrt{2}, -3\sqrt{3})$

Question 2 b)

Center of mass = centroid.

Centroid = where all median lines of a triangle intersect.

$$M_{AB} = \left(\frac{8 + 12}{2}, \frac{12 + 4}{2} \right) = (10, 8)$$

$$m_{M_{AB}C} = \frac{8 - 8}{10 - 2} = 0$$

$$y_{M_{AB}C} = 8 \quad (1)$$

$$M_{BC} = \left(\frac{12 + 2}{2}, 8 + 42 \right) = (7, 6)$$

$$m_{M_{BC}A} = \frac{6 - 12}{7 - 8} = 6$$

$$y_{M_{BC}A} - 12 = 6(x - 8)$$

$$y_{M_{BC}A} = 6x - 48 + 12$$

$$y_{M_{BC}A} = 6x - 36 \quad (2)$$

$$\begin{cases} y_{M_{BC}A} = 8 & (1) \\ y_{M_{BC}A} = 6x - 36 & (2) \end{cases}$$

Sub (1) into (2)

$$8 = 6x - 36$$

$$6x = 44$$

$$x = \frac{44}{6} = \frac{22}{3} \quad (3)$$

By (1), $y = 8$.

∴ The centroid is at $(\frac{22}{3}, 8)$

Question 3

Shortest distance = straight perpendicular line that connects A to a point on line \overline{GH}

$$M_{GH} = \frac{42 + 30}{38 + 16} = \frac{72}{54} = \frac{4}{3}$$

$$M_{\perp GH} = \frac{-3}{4}$$

$$y_{\perp GH} - 32 = \frac{-3}{4}(x + 16)$$

$$y_{\perp GH} = \frac{-3}{4}x + 20 \quad (1)$$

$$y_{GH} + 30 = \frac{4}{3}(x + 16)$$

$$y_{GH} = \frac{4}{3}x - \frac{26}{3} \quad (2)$$

$$\begin{cases} y_{\perp GH} = \frac{-3}{4}x + 20 & (1) \\ y_{GH} = \frac{4}{3}x - \frac{26}{3} & (2) \end{cases}$$

Sub (1) into (2)

$$\frac{-3}{4}x + 20 = \frac{4}{3}x - \frac{26}{3}$$

$$-9x + (12)20 = 16x - 4(26)$$

$$25x = 344$$

$$x = \frac{344}{25} \quad (3)$$

Sub (3) into (1)

$$y = \frac{-3}{4}\left(\frac{344}{25}\right) + 20$$

$$y = \frac{-258}{25} + 20$$

$$y = \frac{-257}{25} + \frac{500}{25}$$

$$y = 24225$$

$$\text{Distance} = \sqrt{\left(-16 - \frac{344}{25}\right)^2 + \left(32 - \frac{242}{25}\right)^2} = 37.2$$

∴ The shortest length pipe is 37.2 units.

Question 4

Let (x, y) be the center of the circle, and r be the radius of the circle.

$$\begin{cases} (x - 4)^2 + (y - 8)^2 = r^2 & (1) \\ (x - 5)^2 + (y - 1)^2 = r^2 & (2) \\ (x + 2)^2 + y^2 = r^2 & (3) \end{cases}$$

Sub (1) into (2)

$$x^2 + 8x + 16 + y^2 - 16y + 64 = x^2 - 10x + 25 + y^2 - 2y + 1$$

$$-8x - 16y + 80 = -10x - 2y + 26$$

$$2x - 14y = -54$$

$$x - 7y = -27 \quad (4)$$

Sub (2) into (3)

$$x^2 + 10x + 25 + y^2 - 2y + 1 = x^2 + 4x + 4 + y^2$$

$$10x - 2y + 26 = 4x + 4$$

$$14x + 2y = 22$$

$$7x + y = 11$$

$$y = 11 - 7x \quad (5)$$

Sub (5) into (4)

$$x - 7(11 - 7x) = -27$$

$$x - 77 + 49x = 27$$

$$50x = 50$$

$$x = 1 \quad (6)$$

Sub (6) into (5)

$$y = 11 - 7(1)$$

$$y = 4 \quad (7)$$

Sub (6), (7) into (3)

$$(1 + 2)^2 + 4^2 = r^2$$

$$r^2 = 16 + 9$$

$$r^2 = 25$$

$\therefore (x - 1)^2 + (y - 4)^2 = 25$ is the equation of the circle.