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James Su authored just now

37950727

Quadratic Equations.md 5.23 KB

Question 1 a)

The zeroes of a quadratic equation are the solutions of x when $ax^2 + bx + c = 0$. The roots of the quadratic equation is when $(x + r_1)(x + r_2) = 0$, more commonly described by the formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Therefore the roots of a quadratic equation are also the zeroes of the quadratic equation.

Question 1 b)

$D = b^2 - 4ac$

If $D = 0$, there is one zero.

$\therefore n^2 - 4(1)(4) = 0$

$n^2 - 16 = 0$

$(n + 4)(n - 4) = 0$

$n = \pm 4$

Question 1 c)

Let h be the height and b be the base. $h = 2b + 4$.

$\therefore (2b + 4)(b) = 168(2)$

$4b^2 + 8b = 168(2)$

$b^2 + 2b = 84 = 0$

$b = \frac{-2 \pm \sqrt{4 + 4(84)}}{4}$

There was an error rendering this math block

$b = -1 + \sqrt{85}$

Question 2 a)

If the quadratic is in $ax^2 + bx + c$, the AOS (axis of symmetry) is at $\frac{-b}{2a}$. And you can plug that value into the quadratic equation to get your optimal value, which is:

$= a(\frac{-b}{2a})^2 + b(\frac{-b}{2a}) + c$

$= \frac{b^2}{4a} + \frac{-b^2}{2a} + c$

$= \frac{-b^2}{4a} + c$

Question 2 b)

$2x^2 + 5x - 1 = 0$

$2(x^2 + \frac{5}{2}x + (\frac{5}{4})^2 - (\frac{5}{4})^2) - 1 = 0$

$2(x + \frac{5}{4})^2 - \frac{25}{8} - 1 = 0$

$2(x + \frac{5}{4})^2 - \frac{33}{8} = 0$

$(x + \frac{5}{4})^2 = \frac{33}{16}$

$x = \pm\sqrt{\frac{33}{16}} - \frac{5}{4}$

$x = \frac{\pm\sqrt{33}}{4} - \frac{5}{4}$

$x = \frac{\sqrt{33} - 5}{4}$ or $\frac{-\sqrt{33} - 5}{4}$

Question 2 c)

Let w be the width between the path and flowerbed, x be the length of the whole rectangle and y be the whole rectangle (flowerbed + path).

$x = 9 + 2w$

$y = 6 + 2w$

$(6 + 2w)(9 + 2w) - (6)(9 = (6)(9)$

$54 + 12w + 18w + 4w^2 = 2(54)$

$4w^2 + 30w = 54$

$2w^2 + 15w - 27 = 0$

$(2w - 3)(w + 9) = 0$

$w = \frac{3}{2}, -9$

$\because w > 0$

$\therefore w = \frac{3}{2}$

$\therefore x = 9 + 3 = 12$

$\therefore y = 6 + 3 = 9$

$P = 2(x + y) \implies P = 2(12 + 9) \implies P = 42$

$\therefore$ The perimeter is $42m$

Question 3 a)

Use discriminant, where $D = b^2 - 4ac$.

$$\begin{cases} \text{If } D > 0 & \text{Then there are 2 real distinct solutions} \\ \text{If } D = 0 & \text{Then there is 1 real solution} \\ \text{If } D < 0 & \text{Then there are no real solutions} \end{cases}$$

Question 3 b)

$y = 12x^2 - 5x - 2$

$y = (3x - 2)(4x + 1)$

$\therefore$ The x-intercepts are at $\frac{2}{3}, \frac{-1}{4}$

Question 3 c)

When $P(x) = 0$, that means it is the break-even point for a value of x (no profit, no loss).

$2k^2 + 12k - 10 = 0 \implies k^2 - 6k + 5 = 0$

$(k - 5)(k - 1) = 0$

$k = 5, 1$

Either 5000 or 1000 rings must be produced so that there is no prodift and no less.

AOS (axis of symmetry) = $\frac{-b}{2a} = \frac{6}{2} = 3$

$\therefore$ 3000 rings should be made to achieve the optimal value.

Maximum profit $= -2(3)^2 + 12(3) - 10$

$= -18 + 30 - 10$

$= 8$

$\therefore 8000$ dollars is the maximum profit.

Question 4 a)

$5x(x-1)+5 = 7+x(1-2x)$

$5x^2-5x = 2+x-2x^2$

$7x^2-6x-2=0$

$x = \frac{6 \pm \sqrt{92}}{14} = \frac{3 \pm \sqrt{23}}{7}$

Question 4 b).

$\because \frac{1}{3}$ and $\frac{-2}{3}$ are the roots of a quadratic equation, that must mean that $(x-\frac{1}{3})(x-\frac{2}{3})$ is a quadratic equation that gives those roots.

After expanding we get:

$y = x^2 - \frac{2}{3}x - \frac{1}{3}x + \frac{2}{3}$

$y = x^2 - x + \frac{2}{3}$

Now we complete the square.

$y = x^2 - x + (\frac{1}{2})^2 - (\frac{1}{2})^2 + \frac{2}{3}$

$y = (x-\frac{1}{2})^2 - \frac{1}{4} + \frac{2}{3}$

$y = (x-\frac{1}{2})^2 + \frac{5}{12}$

Qustion 4 c)

When $h = 0$, the ball hits ground, so:

$-3.2t^2 + 12.8 + 1 = 0$

$t = \frac{12.8 \pm \sqrt{151.04}}{6.4}$

$\because t \geq 0$

$\therefore t = \frac{12.8 \pm \sqrt{151.04}}{6.4}$

$\therefore t \approx 3.9$

The ball will strike the ground at approximately 3.9 seconds.

Question 5 a)

$128 = 96t - 16t^2$

$16t^2 - 96t + 128 = 0$

$t^2 - 6t + 8 = 0$

$(t-2)(t-4)$, at seconds 2 and 4, the rocket reaches $128m$.

Question 5 b)

Break even is when revenue = cost.

$\therefore R(d) = C(d)$

$-40d^2 + 200d = 300 - 40d$

$40d^2 - 240d + 300 = 0$

$2d^2 - 12d + 15 = 0$

$d = \frac{12 \pm \sqrt{24}}{4}$

$$d = \frac{6 \pm \sqrt{6}}{2}$$

At $d = 4.22474407$ or 1.775255135 is when you break even.

Question 5 c)

Let the quation be $y = a(x-d)^2 + c$

Since we know that that $(0,0)$ and $(6,0)$ are the roots of this equation, the AOS is when $x = 3$

$\therefore y = a(x-3)^2 + c.$

Since we know that $(0,0)$ is a point on the parabola, we can susbsitute it into our equation.

$0 = 9a + c \quad (1)$

Since we know that $(4,5)$ is also a point on the parabola, we can susbsitute it int our equation as well.

$5 = a + c \quad (2)$

$$\begin{cases} 9a + c = 0 & (1) \\ a + c = 5 & (2) \end{cases}$$

$(2) - (1)$

$-8a = 5 \implies a = \frac{-5}{8} \quad (3)$

Sub 3 into (2):

$5 = \frac{-5}{8} + c \implies c = \frac{45}{8}$

$\therefore$ Our equation is $y = \frac{-5}{8}(x-3)^2 + \frac{45}{8}$