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Unit 1: Analytical Geometry.md 4.66 KB

Analytical Geometry

Question 1 a)

Lets first find each of the side lengths to determine if the triangle is **obtuse**, **acute** or scalene.

$\overline{AB} = \sqrt{(-1-7)^2+(5-2)^2} = \sqrt{64+9} = \sqrt{73}$

$\overline{BC} = \sqrt{(7-(-1))^2+(2-(-4))^2} = \sqrt{64+36} = \sqrt{100} = 10$

$\overline{AC} = \sqrt{(-1-(-1))^2+(5-(-4))^2} = \sqrt{0^2+9^2} = \sqrt{81} = 9$

$\because \overline{AB} \neq \overline{BC} \neq \overline{AC}$

$\therefore \triangle ABC$ is a scalene triangle.

Question 1 b)

The `orthocenter` is the POI of the heights of a triangle.

$m_{AB} = \dfrac{2-5}{7-(-1)} = \dfrac{-3}{8}$

$m_{\perp AB} = \dfrac{8}{3}$

$y_{\perp AB} - (-4) = \dfrac{8}{3}(x-(-1)) \implies y_{perpAB} + 4 = \dfrac{8}{3}(x+1)$

$y_{\perp AB} = \dfrac{8}{3}x + \dfrac{8}{3} - 4$

$y_{\perp AB} = \dfrac{8}{3}x - \dfrac{4}{3} \quad (1)$

$m_{BC} = \dfrac{2-(-4)}{7-(-1)} = \dfrac{6}{8} = \dfrac{3}{4}$

$m_{\perp BC} = \dfrac{-4}{3}$

$y_{\perp BC} - 5 = \dfrac{-4}{3}(x-(-1)) \implies y_{\perp BC} - 5 = \dfrac{-4}{3}(x+1)$

$y_{\perp BC} = \dfrac{-4}{3}x - \dfrac{4}{3} + 5$

$y_{\perp BC} = \dfrac{-4}{3}x + \dfrac{11}{3}$

$$\begin{cases} y_{\perp AB} = \dfrac{8}{3}x - \dfrac{4}{3} & (1) \\ y_{\perp BC} = \dfrac{-4}{3}x + \dfrac{11}{3} & (2) \end{cases}$$

Sub (1) into (2):

$\dfrac{8}{3}x - \dfrac{4}{3} = \dfrac{-4}{3} + \dfrac{11}{3}$

$8x - 4 = -4x + 11$

$12x = 15$

$x = \dfrac{5}{4} \quad (3)$

Sub (3) into (2)

$y = \frac{-20}{12} + \frac{11}{3}$

$y = \frac{-5}{3} + \frac{11}{3}$

$y = \frac{6}{3} = 2$

$y = 2$

$\therefore$ The `orthocenter` is at $(\frac{5}{4}, 2)$

Question 2 a)

midpoint = $(\frac{\sqrt{72}+\sqrt{32}}{2}, \frac{-\sqrt{12}-\sqrt{48}}{2})$

$= (\frac{6\sqrt{2}+4\sqrt{2}}{2}, \frac{-2\sqrt{3}, -4\sqrt{3}}{2})$

$= 3\sqrt{2}+2\sqrt{2}, -\sqrt{3}-2\sqrt{3}$

$= (5\sqrt{2}, -3\sqrt{3})$

$\therefore$ The midpoint is at $(5\sqrt{2}, -3\sqrt{3})$

Question 2 b)

Center of mass = centroid.

Centroid = where all median lines of a trinagle intersect.

$M_{AB} = (\frac{8+12}{2}, \frac{12+4}{2}) = (10, 8)$

$m_{M_{AB}C} = \frac{8-8}{10-2} = 0$

$y_{M_{AB}C} = 8 \quad (1)$

$M_{BC} = (\frac{12+2}{2}, 8+42) = (7, 6)$

$m_{M_{BC}A} = \frac{6-12}{7-8} = 6$

$y_{M_{BC}A} - 12 = 6(x-8)$

$y_{M_{BC}A} = 6x - 48 + 12$

$y_{M_{BC}A} = 6x - 36 \quad (2)$

$$\begin{cases} y_{M_{BC}A} = 8 & (1) \\ y_{M_{BC}A} = 6x - 36 & (2) \end{cases}$$

Sub (1) into (2)

$8 = 6x - 36$

$6x = 44$

$x = \frac{44}{6} = \frac{22}{3} \quad (3)$

By (1), $y = 8$.

$\therefore$ The centroid is at $(\frac{22}{3}, 8)$

Question 3

Shortest distance = straight perpendicular line that connets A to a point on line $\overline{GH}$

$M_{GH} = \frac{42+30}{38+16} = \frac{72}{54} = \frac{4}{3}$

$M_{\perp GH} = \frac{-3}{4}$

$y_{\perp GH} - 32 = \frac{-3}{4}(x+16)$

$y_{\perp GH} = \frac{-3}{4}x + 20 \quad (1)$

$y_{GH} + 30 = \frac{4}{3}(x + 16)$

$y_{GH} = \frac{4}{3}x - \frac{26}{3} \quad (2)$

$$\begin{cases} y_{\perp GH} = \frac{-3}{4}x + 20 & (1) \\ y_{GH} = \frac{4}{3}x - \frac{26}{3} & (2) \end{cases}$$

Sub (1) into (2)

$\frac{-3}{4}x + 20 = \frac{4}{3}x - \frac{26}{3}$

$-9x + (12)20 = 16x - 4(26)$

$25x = 344$

$x = \frac{344}{25} \quad (3)$

Sub (3) into (1)

$y = \frac{-3}{4}(\frac{344}{25}) + 20$

$y = \frac{-258}{25} + 20$

$y = \frac{-257}{25} + \frac{500}{25}$

$y = 24225$

Distance $= \sqrt{(-16 - \frac{344}{25})^2 + (32 - \frac{242}{25})^2} = 37.2$

$\therefore$ The shortest length pipe is 37.2 units.

Question 4

Let (x, y) be the center of the circle, and r be the radius of the circle.

$$\begin{cases} (x-4)^2 + (y-8)^2 = r^2 & (1) \\ (x-5)^2 + (y-1)^2 = r^2 & (2) \\ (x+2)^2 + y^2 = r^2 & (3) \end{cases}$$

Sub (1) into (2)

$x^2 - 8x + 16 + y^2 - 16y + 64 = x^2 - 10x + 25 + y^2 - 2y + 1$

$-8x - 16y + 80 = -10x - 2y + 26$

$2x - 14y = -54$

$x - 7y = -27 \quad (4)$

Sub (2) into (3)

$x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 + 4x + 4 + y^2$

$-10x - 2y + 26 = 4x + 4$

$14x + 2y = 22$

$7x + y = 11$

$y = 11 - 7x \quad (5)$

Sub (5) into (4)

$x - 7(11 - 7x) = -27$

$x - 77 + 49x = 27$

$50x = 50$

$x = 1 \quad (6)$

Sub (6) into (5)

$y = 11 - 7(1)$

$y = 4 \quad (7)$

Sub $(6), (7)$ into (3)

$(1 + 2)^2 + 4^2 = r^2$

$r^2 = 16 + 9$

$r^2 = 25$

$\therefore (x - 1)^2 + (y - 4)^2 = 25$ is the equation of the circle.